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WELL-POSEDNESS AND SPECTRAL ESTIMATION FOR INFINITE  
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STATE UNIV BLACKSBURG DEPT OF M. C A BEATTIE ET AL.

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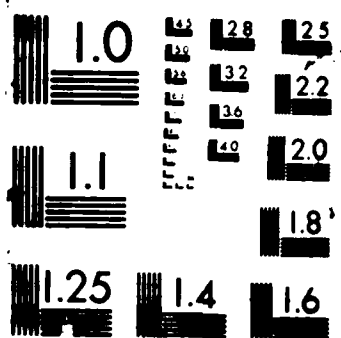
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<p>To develop a mathematical model for the motions of an airfoil, with flap, in a two dimensional unsteady flow of an inviscid, incompressible fluid. We establish necessary and sufficient conditions for the well-posedness for a large class of functional differential equations containing those used to model the aeroelastic system. Significant progress has been made in developing efficient numerical approaches for resolving intermediate problems. Further work has provided refined rate estimates for the closure of spectral estimates and a formulation for resonances of sparse frame structures has been developed and tested.</p>			
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**WELL-POSEDNESS AND SPECTRAL ESTIMATION  
FOR INFINITE DIMENSIONAL SYSTEMS**

**Final Report: AFOSR-84-0326**

**C. A. BEATTIE AND T. L. HERDMAN**

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The research described below was completed during the period September 1984 to September 1987. This work was funded by the Air Force Office of Scientific Research under grant AFOSR-84-0326.

### Aeroelastic Systems

Our investigation of aeroelastic systems focused on the development of a mathematical model for the motions of an airfoil, with flap, in a two-dimensional, unsteady flow of an inviscid, incompressible fluid. We have shown that the evolution equation for the circulation on the airfoil can be coupled to the rigid-body dynamics of the airfoil to obtain a complete set of functional differential equations that describe the composite system. This system belongs to a general class of neutral functional differential equations which have a nonatomic difference operator. Our investigation of the aeroelastic system resulted in establishing necessary and sufficient conditions for the well-posedness for a large class of functional differential equations (finite delay). This class of equations contains the "standard" neutral and retarded functional differential equations as well as many weakly singular integro-differential equations. Using the theory that was developed for this general class of functional differential equations, we have shown that our finite delay approximation for the aeroelastic system is well-posed on the product space  $\mathbb{R}^8 \times L_p(-r, 0)$  for  $p > 1$  if and only if  $p < 2$ .

Our study of the aeroelastic system (finite delay) raises an interesting question concerning the derivation of the equations of motion. Almost all derivations of these equations arrive at the circulation equation by using the Söhngen inversion formula. The validity of this inversion formula can be assured only in the case that the circulation history is assumed to belong in  $L_p$  for  $p > 2$ . However, we have noted that in general the resulting model is not well-posed in the space of  $L_p$  circulation histories. Therefore, if one restricts the space of circulation histories to be contained in some  $L_p$  space with  $p > 2$  (in order to use the Söhngen inversion formula), then the resulting neutral functional differential equation will not be well-posed in that space. On the other hand, if one formally applies this inversion formula then the resulting system will be well-posed provided that the initial data lies in  $\mathbb{R}^8 \times L_p(-r, 0)$  with  $p < 2$ . At this time it is not known if a general

inversion formula for Hilbert transform can be used in place of the Söhngen inversion formula in order to avoid the restriction,  $p > 2$ .

It is to be noted that the full set equations for the aeroelastic system involve infinite delays. The results for the general class of functional differential equation (finite delay) described above should be extended to the infinite delay problem. It appears that many of our results can be extended to the infinite delay problem by using a proper weighting on the corresponding function space.

### Spectral Estimation

Significant progress has been made in developing efficient numerical approaches to resolving intermediate problems. This has resulted in a novel combination of spectrum slicing and Rayleigh quotient iteration having both rapid cubic local convergence and favorable global behavior. We have additionally completed a thorough study of the asymptotic convergence behavior of intermediate problem methods and found quadratic closure in gap that provides a similar asymptotic rate as the Rayleigh-Ritz (finite element) method yet allows relatively unbounded perturbations to be considered for the first time.

One of the most substantial areas of progress is in the evaluation of lower bounds for the resonant frequencies of nonuniform space frames. The method we have developed is a variant of the exact displacement method and can provide very high accuracy eigenvalue estimates for far less effort than finite-element methods require. Although our computational formulation involves a highly nonlinear root-finding problem (vs. the highly structured linear problem that finite-element formulations produce), very high accuracy for higher modes is generally obtained.

### Interaction with Air Force Laboratories

During the period of this grant, Dr. Herdman has consulted with Air Force Flight Dynamics Laboratory personnel for motivation and guidance for this research effort.

### Research Articles

The research associated with this grant has produced the following papers:

Beattie, C., An extension of Aronszajn's rule: Slicing the spectrum for intermediate problems, *SIAM J. Num. Anal.*, 24 (4), pp. 828-843.

Beattie, C., Lower bounds for the resonant frequencies of nonuniform frame structures, *J. Appl. Math. Mech. (ZAMM)*, to appear.

Beattie, C. and Banach, A., Rapid resolution of truncated intermediate problems, in Eigenvalue Problems in Engineering Science and their Numerical Treatment (eds: J. Albrecht, L. Collatz, W. Velte, W. Wunderlich), ISNM Series, Birkhauser, to appear.

Beattie, C. and Greenlee, W.M., Convergence rates for intermediate problems, *Manuscripta Mathematica*, to appear.

Beattie, C., Boisen, M., and Johnson, L., Inertia-preserving secant updates, *J. Opt. Th. and Appl. (JOTA)*, to appear.

Beattie, C. and Greenlee, W.M., Improved Convergence rates for intermediate problems, submitted.

J.A. Burns, T.L. Herdman and J. Turi, Well-Posedness of functional differential equations with nonatomic  $D$  operator, Trends in theory and Practice of Nonlinear Analysis, V. Lakshmikantham, Ed., Academic Press, 1985, 71-77.

J.A. Burns, E.M. Cliff, T.L. Herdman and J. Turi, On integral transforms appearing in the derivation of the equations of an aeroelastic system, Nonlinear Analysis and Applications, V. Lakshmikantham, Ed., in press, pp. 89-98.

J.A. Burns, T.L. Herdman and J. Turi, Nonatomic neutral functional differential equations, Nonlinear Analysis and Applications, V. Lakshmikantham, Ed., in press.

J.A. Burns, T.L. Herdman and J. Turi, Neutral functional differential equations with weakly singular kernels, submitted.

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